

Negative parity nonstrange baryons in large N_c QCD: quark excitation versus meson-nucleon scattering

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Abstract

We show that the two complementary pictures of large N_c baryons - the single-quark orbital excitation about a symmetric core and the meson-nucleon resonance - are compatible for $\ell = 3$ SU(4) baryons. The proof is based on a simple Hamiltonian including operators up to order $\mathcal{O}(N_c^0)$ used previously in the literature for $\ell = 1$.

1 The status of the $1/N_c$ expansion method

The large N_c QCD, or alternatively the $1/N_c$ expansion method, proposed by 't Hooft [1] and implemented by Witten [2] became a valuable tool to study baryon properties in terms of the parameter $1/N_c$ where N_c is the number of colors. According to Witten's intuitive picture, a baryon containing N_c quarks is seen as a bound state in an average self-consistent potential of a Hartree type and the corrections to the Hartree approximation are of order $1/N_c$.

Ten years after 't Hooft's work, Gervais and Sakita [3] and independently Dashen and Manohar in 1993 [4] derived a set of consistency conditions for the pion-baryon coupling constants which imply that the large N_c limit of QCD has an exact contracted $SU(2N_f)_c$ symmetry when $N_c \rightarrow \infty$, N_f being the number of flavors. For ground state baryons the $SU(2N_f)$ symmetry is broken by corrections proportional to $1/N_c$ [5, 6].

Analogous to s-wave baryons, consistency conditions which constrain the strong couplings of excited baryons to pions were derived in Ref. [7]. These consistency conditions

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predict the equality between pion couplings to excited states and pion couplings to s-wave baryons. These predictions are consistent with the nonrelativistic quark model.

A few years later, in the spirit of the Hartree approximation a procedure for constructing large N_c baryon wave functions with mixed symmetric spin-flavor parts has been proposed [8] and an operator analysis was performed for $\ell = 1$ baryons [9]. It was proven that, for such states, the $SU(2N_f)$ breaking occurs at order N_c^0 , instead of $1/N_c$, as it is the case for ground and also symmetric excited states $[56, \ell^+]$ (for the latter see Refs. [10, 11]). This procedure has been extended to positive parity nonstrange baryons belonging to the $[70, \ell^+]$ with $\ell = 0$ and 2 [12]. In addition, in Ref. [12], the dependence of the contribution of the linear term in N_c , of the spin-orbit and of the spin-spin terms in the mass formula was presented as a function of the excitation energy or alternatively in terms of the band number N . Based on this analysis an impressive global compatibility between the $1/N_c$ expansion and the quark model results for $N = 0, 1, 2$ and 4 [13] was found (for a review see Ref. [14]). More recently the $[70, 1^-]$ multiplet was reanalyzed by using an exact wave function, instead of the Hartree-type wave function, which allowed to keep control of the Pauli principle at any stage of the calculations [15]. The novelty was that the isospin-isospin term, neglected previously [9] becomes as dominant in Δ resonances as the spin-spin term in N^* resonances.

The purpose of this work is to analyze the compatibility between the $1/N_c$ expansion method in the so-called *quark – shell picture* and the *resonance or scattering picture* defined in the framework of chiral soliton models. Details can be found in Ref. [16].

2 Negative parity baryons

If an excited baryon belongs to a symmetric $[56]$ -plet the three-quark system can be treated similarly to the ground state in the flavour-spin degrees of freedom, but one has to take into account the presence of an orbital excitation in the space part of the wave function [10, 11]. If the baryon state is described by a mixed symmetric representation, $[70]$ in $SU(6)$ notation, the treatment becomes more complicated. In particular, the resonances up to 2 GeV belong to $[70, 1^-]$, $[70, 0^+]$ or $[70, 2^+]$ multiplets and beyond to 2 GeV to $[70, 3^-]$, $[70, 5^-]$, etc.

In the following we adopt the standard way to study the $[70]$ -plets which, as already mentioned, is related to the Hartree approximation [8]. An excited baryon is described by a symmetric core plus an excited quark coupled to this core, see *e.g.* [9, 12, 17, 18]. The core is treated in a way similar to that of the ground state. In this method each $SU(2N_f) \times O(3)$ generator is separated into two parts

$$S^i = s^i + S_c^i; \quad T^a = t^a + T_c^a; \quad G^{ia} = g^{ia} + G_c^{ia}; \quad \ell^i = \ell_q^i + \ell_c^i, \quad (1)$$

where s^i , t^a , g^{ia} and ℓ_q^i are the excited quark operators and S_c^i , T_c^a , G_c^{ia} and ℓ_c^i the corresponding core operators.

2.1 The quark-shell picture

In the quark-shell picture we use the procedure of Ref. [19], equivalent to that of Ref. [20], later extended in Ref. [21]. We start from the leading-order Hamiltonian including operators up to order $\mathcal{O}(N_c^0)$ which has the following form

$$H = c_1 \mathbb{1} + c_2 \ell \cdot s + c_3 \frac{1}{N_c} \ell^{(2)} \cdot g \cdot G_c \quad (2)$$

This operator is defined in the spirit of a Hartree picture (mean field) where the matrix elements of the first term are proportional to N_c on all baryons [2]. The spin-orbit term $\ell \cdot s$ which is a one-body operator and the third term - a two-body operator containing the tensor $\ell^{(2)ij}$ of $O(3)$ - have matrix elements of order $\mathcal{O}(N_c^0)$. The neglect of $1/N_c$ corrections in the $1/N_c$ expansion makes sense for the comparison with the scattering picture in the large N_c limit, described in the following section.

One can see that the Hamiltonian (2) reproduces the characteristic N_c scaling for the excitation energy of baryons which is N_c^0 [2].

2.1.1 The nucleon case

In large N_c the color part of the wave function is antisymmetric so that the orbital-spin-flavor part must be symmetric to satisfy the Pauli principle. A quanta of orbital excitation requires the orbital part to be mixed symmetric, the lowest state having the partition $[N_c - 1, 1]$. We have the following $[N_c - 1, 1]$ spin-flavor (SF) states which form a symmetric state with the orbital $\ell = 3$ state of partition $[N_c - 1, 1]$

1. $[N_c - 1, 1]_{SF} = \left[\frac{N_c+1}{2}, \frac{N_c-1}{2} \right]_S \times \left[\frac{N_c+1}{2}, \frac{N_c-1}{2} \right]_F, N_c \geq 3$
with $S = 1/2$ and $J = 5/2, 7/2$
2. $[N_c - 1, 1]_{SF} = \left[\frac{N_c+3}{2}, \frac{N_c-3}{2} \right]_S \times \left[\frac{N_c+1}{2}, \frac{N_c-1}{2} \right]_F, N_c \geq 3$
with $S = 3/2$ and $J = 3/2, 5/2, 7/2, 9/2$.

They give rise to matrices of a given J either 2×2 or 1×1 depending on the multiplicity of J . States of symmetry $[N_c - 1, 1]_{SF}$ with $S = 5/2$, like for Δ (see below), which together with $\ell = 3$ could give rise to $J = 11/2$, are not allowed for N , by inner products of the permutation group [22]. Therefore the experimentally observed resonance $N(2600)I_{11/2}$ should belong to the $N = 5$ band ($\ell = 5$). For $N_c = 3$ the above states correspond to the 2_8 and 4_8 multiplets of $SU(2) \times SU(3)$ respectively.

2.1.2 The Δ case

In this case the Pauli principle allows the following states

1. $[N_c - 1, 1]_{SF} = \left[\frac{N_c+1}{2}, \frac{N_c-1}{2} \right]_S \times \left[\frac{N_c+3}{2}, \frac{N_c-3}{2} \right]_F, N_c \geq 3$
with $S = 1/2$ and $J = 5/2, 7/2$,

2. $[N_c - 1, 1]_{SF} = \left[\frac{N_c+3}{2}, \frac{N_c-3}{2} \right]_S \times \left[\frac{N_c+3}{2}, \frac{N_c-3}{2} \right]_F$, $N_c \geq 5$
with $S = 3/2$ and $J = 3/2, 5/2, 7/2, 9/2$,
3. $[N_c - 1, 1]_{SF} = \left[\frac{N_c+5}{2}, \frac{N_c-5}{2} \right]_S \times \left[\frac{N_c+3}{2}, \frac{N_c-3}{2} \right]_F$, $N_c \geq 7$
with $S = 5/2$ and $J = 1/2, 3/2, 5/2, 7/2, 9/2, 11/2$.

As above, they indicate the size of a matrix of fixed J for the Hamiltonian (2). For example, the matrix of $\Delta_{5/2}$ is 3×3 , because all three states can have $J = 5/2$. For $N_c = 3$ the first state belongs to the ${}^2 10$ multiplet. The other two types of states do not appear in the real world with $N_c = 3$. Note that both for N_J and Δ_J states the size of a given matrix equals the multiplicity of the corresponding state indicated in Table 1 of Ref. [21] for $\ell = 3$.

The Hamiltonian (2) is diagonalized in the bases defined above. Let us denote the eigenvalues either by $m_{N_J}^{(i)}$ or $m_{\Delta_J}^{(i)}$ with $i = 1, 2$ or 3 , depending on how many eigenvalues are at a fixed J . The Hamiltonian has analytical solutions, all eigenvalues being linear functions in the coefficients c_1 , c_2 and c_3 . It is remarkable that the 18 available eigenstates with $\ell = 3$ fall into three degenerate multiplets, like for $\ell = 1$. If the degenerate masses are denoted by m'_2 , m_3 and m_4 we have

$$m'_2 = m_{\Delta_{1/2}}^{(1)} = m_{N_{3/2}}^{(1)} = m_{\Delta_{3/2}}^{(1)} = m_{N_{5/2}}^{(1)} = m_{\Delta_{5/2}}^{(1)} = m_{\Delta_{7/2}}^{(1)}, \quad (3)$$

$$m_3 = m_{\Delta_{3/2}}^{(2)} = m_{N_{5/2}}^{(2)} = m_{\Delta_{5/2}}^{(2)} = m_{N_{7/2}}^{(1)} = m_{\Delta_{7/2}}^{(2)} = m_{\Delta_{9/2}}^{(1)}, \quad (4)$$

$$m_4 = m_{\Delta_{5/2}}^{(3)} = m_{N_{7/2}}^{(2)} = m_{\Delta_{7/2}}^{(3)} = m_{N_{9/2}}^{(1)} = m_{\Delta_{9/2}}^{(2)} = m_{\Delta_{11/2}}^{(1)}, \quad (5)$$

where

$$m'_2 = c_1 N_c - 2c_2 - \frac{3}{4}c_3, \quad (6)$$

$$m_3 = c_1 N_c - \frac{1}{2}c_2 + \frac{15}{16}c_3, \quad (7)$$

$$m_4 = c_1 N_c + \frac{3}{2}c_2 - \frac{5}{16}c_3. \quad (8)$$

The notation m'_2 is used to distinguish this eigenvalue from m_2 of Ref. [19].

In the following subsection we shall see that the scattering picture gives an identical pattern of degeneracy in the quantum numbers, but the resonance mass is not quantitatively defined. Therefore only a qualitative compatibility can be established.

2.2 The meson-nucleon scattering picture

Here we are concerned with nonstrange baryons, as above, and look for a degeneracy pattern in the resonance picture. The starting point in this analysis are the linear relations of the S matrices $S_{LL'RR'IJ}^\pi$ and $S_{LL'RR'IJ}^\eta$ of π and η scattering off a ground state baryon in terms of K -amplitudes. They are given by the following equations [19, 21]

$$S_{LL'RR'IJ}^\pi = \sum_K (-1)^{R'-R} \sqrt{(2R+1)(2R'+1)(2K+1)} \begin{Bmatrix} K & I & J \\ R' & L' & 1 \end{Bmatrix} \begin{Bmatrix} K & I & J \\ R & L & 1 \end{Bmatrix} s_{KLL'}^\pi, \quad (9)$$

and

$$S_{LRJ}^\eta = \sum_K \delta_{KL} \delta(LRJ) s_K^\eta, \quad (10)$$

where $s_{KL'L}^\pi$ and s_K^η are the reduced amplitudes. The notation is as follows. For π scattering R and R' are the spin of the incoming and outgoing baryons respectively ($R=1/2$ for N and $R=3/2$ for Δ), L and L' are the partial wave angular momentum of the incident and final π respectively (the orbital angular momentum L of η remains unchanged), I and J represent the total isospin and total angular momentum associated to a given resonance and K is the magnitude of the *grand spin* $\vec{K} = \vec{I} + \vec{J}$. The $6j$ coefficients imply four triangle rules $\delta(LRJ)$, $\delta(R1I)$, $\delta(L1K)$ and $\delta(IJK)$.

These equations were first derived in the context of the chiral soliton model [23, 24] where the mean-field breaks the rotational and isospin symmetries, so that J and I are not conserved but the *grand spin* K is conserved and excitations can be labelled by K . These relations are exact in large N_c QCD and are independent of any model assumption.

The meaning of Eq. (9) is that there are more amplitudes $S_{LL'RR'IJ}^\pi$ than there are $s_{KLL'}^\pi$ amplitudes. The reason is that the IJ as well as the RR' dependence is contained only in the geometrical factor containing the two $6j$ coefficients. Then, for example, in the πN scattering, in order for a resonance to occur in one channel there must be a resonance in at least one of the contributing amplitudes $s_{KLL'}^\pi$. But as $s_{KLL'}^\pi$ contributes in more than one channel, all these channels resonate at the same energy and this implies degeneracy in the excited spectrum. From the chiral soliton model there is no reason to suspect degeneracy between different K sectors.

From the meson-baryon scattering relations (9) and (10) three sets of degenerate states have been found for $\ell = 1$ orbital excitations [19]. There is a clear correspondence between these sets and the three towers of states [19, 20] of the excited quark picture provided by the symmetric core + excited quark scheme [9]. They correspond to $K = 0, 1$ and 2 in the resonance picture. But the resonance picture also provides a $K = 3$ due to the amplitude s_{322}^π . As this is different from the other $s_{KL'L}^\pi$, in Ref. [19] it was interpreted as belonging to the $N = 3$ band.

Here we extend the work of Ref. [19, 21] to $\ell = 3$ excited states which belong to the $N = 3$ band. The partial wave amplitudes of interest and their expansion in terms of K -amplitudes from Eqs. (9) and (10) can be found in Tables I-III of Ref. [16]. They correspond to $L = L' = 2$, $L = L' = 4$ and $L = L' = 6$ respectively. From those tables one can infer the following degenerate towers of states with their contributing amplitudes

$$\Delta_{1/2}, \quad N_{3/2}, \quad \Delta_{3/2}, \quad N_{5/2}, \quad \Delta_{5/2}, \quad \Delta_{7/2}, \quad (s_{222}^\pi, s_2^\eta), \quad (11)$$

$$\Delta_{3/2}, \quad N_{5/2}, \quad \Delta_{5/2}, \quad N_{7/2}, \quad \Delta_{7/2}, \quad \Delta_{9/2}, \quad (s_{322}^\pi, s_{344}^\pi), \quad (12)$$

$$\Delta_{5/2}, \quad N_{7/2}, \quad \Delta_{7/2}, \quad N_{9/2}, \quad \Delta_{9/2}, \quad \Delta_{11/2}, \quad (s_{444}^\pi, s_4^\eta), \quad (13)$$

$$\Delta_{7/2}, \quad N_{9/2}, \quad \Delta_{9/2}, \quad \Delta_{11/2}, \quad (s_{544}^\pi, s_{566}^\pi), \quad (14)$$

$$\Delta_{9/2}, \quad \Delta_{11/2}, \quad (s_{666}^\pi, s_6^\eta) \quad (15)$$

associated to $K = 2, 3, 4, 5$ and 6 respectively.

We can compare the towers (11)-(15) with the quark-shell model results of (3)-(5). The first observation is that the agreement of (11) ($K = 2$) with (3), of (12) ($K = 3$) with (4) and of (13) ($K = 4$) with (5) is perfect regarding the quantum numbers. Second, we note that the resonance picture can have poles with $K = 5, 6$ which infer the towers (14) and (15). They have no counterpart in the quark-shell picture for $\ell = 3$. But there is no problem because the poles with $K = 5, 6$ can belong to a higher band, namely $N = 5$ ($\ell = 5$) without spoiling the compatibility.

Comparing these results with those of Ref. [21] one can conclude that one can associate a common $K = 2$ to $\ell = 1$ and $\ell = 3$. For this value of K the triangular rule $\delta(K\ell 1)$ proposed in Ref [21] is satisfied. The quark-shell picture brings however more information than the resonance picture due to the fact that it implies an energy dependence via the ℓ dependence which measures the orbital excitation. Note that m'_2 is different from m_2 of $\ell = 1$ [19, 20]. Because in the resonance picture they stem from the same amplitude s_{222}^π , one expects that this amplitude possesses two poles at two distinct energies, in order to have compatibility. Thus the number of poles of the reduced amplitudes s_{KLL}^π remains an open question.

We anticipate that a similar situation will appear for every value of K associated to two distinct values of ℓ , satisfying the $\delta(K\ell 1)$ rule, for example, for $K = 4$ which is common to $\ell = 3$ and $\ell = 5$.

3 Conclusions

We have compared two alternative pictures for baryon resonances consistent with large the N_c QCD limit and found that the two pictures are compatible for $\ell = 3$ excited states, as it was the case for $\ell = 1$. The quark-shell picture is practical and successful in describing known resonances and in predicting other members of the excited octets and decuplets. But the extended symmetry $SU(2N_f) \times O(3)$ where $O(3)$, which is essential to include orbital excitations, does not have a direct link to large N_c . On the other hand the scattering picture is close to experimental analysis but it is not clear where the pole positions should lie. It is however very encouraging that the two pictures give sets of degenerate states with identical quantum numbers when one works at order $\mathcal{O}(N_c^0)$. It is a qualitative proof that the spin-flavor picture is valid and useful for baryon phenomenology.

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